Quantile Composite-based Path Modeling for measuring equitable and sustainable well-being

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Dealing with Complexity in Society: from Plurality of Data to Synthetic Indicators

17th-18th September 2015, Padua (Italy)
Objectives

Background

Measuring well-being
The BES project

Methods

Partial Least Squares Path Model
Quantile Regression
Quantile Correlation
Quantile Composite-based Path Model

Results

Conclusions
Combining PLS Path Modeling and Quantile Regression

Quantile Composite-based Path Modeling (QC-PM)

Application framework:
measuring equitable and sustainable well-being
Aim of the paper

Combining PLS Path Modeling and Quantile Regression
Quantile Composite-based Path Modeling (QC-PM)

Application framework:
measuring equitable and sustainable well-being

The main goals of the paper:

- Regressor effects on the whole dependent variable distribution
- Heteroscedastic relationships
- Presence of outliers
- Skewed dependent variable
Aim of the paper

Methodological framework: multivariate analysis

- PLS Path Modeling (PLSPM)
- Quantile Regression (QR)
- Quantile Correlation (QC)
- Quantile Composite-based Path Modeling (QC-PM)
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Measuring well-being

A short agenda

2007 OECD World Forum: The Istanbul Declaration - measuring the progress of societies in all dimensions to improve policy making, democracy and citizens wellbeing

2007 European Commission: Beyond GDP - Measuring progress, true wealth, and the well-being of nations

2008 Commission on the Measurement of Economic Performance and Social Progress (Stiglitz-Sen-Fitoussi Commission)

2011 ONS starts developing new measures of national well-being

Horizon 2020 states that country performance and best practices must be addressed and monitored

Europe 2020 strategy for smart, sustainable and inclusive growth

...
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Measuring well-being

The BES project

- Italian project to measure equitable and sustainable well-being (BES - www.misuredelbenessere.it)
- National and regional level
- Italian National Institute of Statistics (Istat)
- National Council for Economy and Labour (Cnel)

Provinces BES - 2014

- Measuring BES at local level according to the BES framework
- Nuts3 level
- Italian National Institute of Statistics (Istat)
- National network of the Provinces Statistical Offices (Cuspi)
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The data structure

- **Q** blocks of manifest variables (MVs):
- **Q** latent variables (LVs)

**Model representation**

- \( (X_1) \) with variables: \( x_{11}, x_{21}, x_{31} \)
- \( (X_2) \) with variables: \( x_{12}, x_{22} \)
- \( (X_3) \) with variables: \( x_{13}, x_{23}, x_{33}, x_{43}, x_{53}, x_{63} \)

- \( \xi_1 \) with variable: \( x_{11}, x_{21}, x_{31} \)
- \( \xi_2 \) with variable: \( x_{12}, x_{22} \)
- \( \xi_3 \) with variable: \( x_{13}, x_{23}, x_{33}, x_{43}, x_{53}, x_{63} \)

- \( Q = 3 \)
Partial Least Squares Path Model (PLSPM)

(Wold, 1985) (Tenenhaus et al., 2005) (Esposito Vinzi et al., 2010)
PLSPM: the main steps of the algorithm

1. Data pre-processing
2. Initialization
3. Inner estimation (structural model)
4. Outer estimation (measurement model)
5. Latent score estimation
6. Weight convergence
7. Final output
   1. LV scores
   2. Path coefficients

(Lohmöller 1987, 1989)
Quantile Composite-based Path Model

A quantile approach in the main steps of the PLSPM algorithm

1. Data pre-processing
2. Initialization
3. Inner estimation (structural model)
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(Davino and Vinzi, 2015)
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Classical vs quantile linear regression

Classical linear regression (conditional expected value)

estimation of the conditional mean of a response variable (y) distribution as a function of a set X of predictor variables

\[ E(y | X) = X\beta \]

Quantile regression (conditional quantiles)

estimation of the conditional quantiles of a response variable (y) distribution as a function of a set X of predictor variables

\[ Q_\theta(y | X) = X\beta(\theta) \]

where: \( 0 < \theta < 1 \)
Classical vs quantile linear regression

Classical linear regression
(conditional expected value)

estimation of the conditional mean of a response variable (y) distribution as a function of a set X of predictor variables

\[ E(y \mid X) = X\beta \]

Quantile regression
(conditional quantiles)

estimation of the conditional quantiles of a response variable (y) distribution as a function of a set X of predictor variables

\[ Q_\theta(y \mid X) = X\beta(\theta) \]

where: \( 0 < \theta < 1 \)
**Classical vs quantile linear regression**

**Classical linear regression**
(conditional expected value)

estimation of the conditional mean of a response variable ($y$) distribution as a function of a set $X$ of predictor variables

$$E(y \mid X) = X\beta$$

$$\beta_i = \frac{\partial E(y)}{\partial x_i}$$

**Quantile regression**
(conditional quantiles)

estimation of the conditional quantiles of a response variable ($y$) distribution as a function of a set $X$ of predictor variables

$$Q_\theta(y \mid X) = X\beta(\theta)$$

where: $(0 < \theta < 1)$

$$\beta_i(\theta) = \frac{\partial Q_\theta(y)}{\partial x_i}$$
Quantile Regression (QR)

The conditional quantile estimator

\[ \hat{\beta}(\theta) = \text{arg} \min_{\beta(\theta)} \sum_{i=1}^{n} \rho_{\theta} \left( y_i - x_i^\top \beta(\theta) \right) \]

\[ \rho_{\theta} = \begin{cases} \theta(u) & \text{if } u > 0 \\ (\theta - 1)u & \text{if } u \leq 0 \end{cases} \]

- No parametric distribution assumptions are required for the error distribution
- \( \theta \) regression lines
- 100(1-\( \theta \))% of points above the QR line and 100\( \theta \)% below
- Coefficients inference using a bootstrap approach

(Koenker and Basset, 1978) (Koenker, 2005) (Davino et al., 2013)
A very simple example

- Slopes: rate of change of the $y \theta^{th}$ conditional quantile per unit change of the regressor
- Fitted values reconstruct the conditional quantiles

<table>
<thead>
<tr>
<th></th>
<th>intercept</th>
<th>slope</th>
<th>$\hat{y}(x = 1)$</th>
<th>$\hat{y}(x = 2)$</th>
<th>mean($\hat{y}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>27.13</td>
<td>-7.53</td>
<td>19.60</td>
<td>12.07</td>
<td>17.34</td>
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<tr>
<td>$\theta = 0.1$</td>
<td>6.80</td>
<td>-3.07</td>
<td>3.73</td>
<td>0.66</td>
<td>2.81</td>
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<tr>
<td>$\theta = 0.5$</td>
<td>18.98</td>
<td>-5.71</td>
<td>13.27</td>
<td>7.56</td>
<td>11.55</td>
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<tr>
<td>$\theta = 0.9$</td>
<td>66.56</td>
<td>-18.00</td>
<td>48.56</td>
<td>30.56</td>
<td>43.16</td>
</tr>
</tbody>
</table>
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Quantile Correlation (QC)

**Q covariance**

\[
qcov_\theta \{Y, X \} = \text{cov} \{I(Y - Q_\theta(Y) > 0), X\} = \text{cov} \{I(Y - Q_\theta(Y) > 0), X\} = E \{\psi_\theta [Y - Q_\theta (Y)] [X - E (X)]\}
\]

- Y, X: random variables
- \(\theta \in (0, 1)\): a quantile
- \(I(\cdot)\): indicator function
- \(\psi_\theta (u) = \theta - I(u < 0)\)

**Q correlation (QC)**

\[
qcor_\theta \{Y, X \} = \frac{qcov_\theta \{Y, X \}}{\sqrt{\text{var}\{\psi_\theta [Y - Q_\theta (Y)]\}} \text{var}(X)} = \frac{qcov_\theta \{Y, X \}}{\sqrt{(\theta - \theta^2) \text{var}(X)}}
\]

- \(qcor_\theta \): sample QC
- \(\Omega_1\): asymptotic variance

**QC distribution convergence**

\[
\sqrt{n}(\hat{qcor}_\theta \{Y, X \} - qcor_\theta \{Y, X \}) \rightarrow_d N(0, \Omega_1)
\]

- Li G., Li Y., Tsai C.L. (2015)
  Quantile correlations and quantile autoregressive modeling
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(Davino and Vinzi, 2015)
QC-PM: the inner estimation

Path weighting scheme \((\text{quantile } \theta)\)

\[ Q_\theta \left( \hat{\xi}_j \mid \Xi \rightarrow j \right) = \Xi \rightarrow j \hat{\beta} (\theta) \]

\(\Xi \rightarrow j\): matrix of the predecessor LVs

\(\xi_j\): successor LV \(\Rightarrow\) Quantile Regression

\[ Q_\theta \left( \hat{\xi}_j \mid \Xi \rightarrow j \right) = \Xi \rightarrow j \hat{\beta} (\theta) \]

\(\Xi \rightarrow j\): matrix of the predecessor LVs

\(\xi_j\): successor LV \(\Rightarrow\) Quantile Regression

\[ Q_\theta \left( \hat{\xi}_j \mid \Xi \rightarrow j \right) = \Xi \rightarrow j \hat{\beta} (\theta) \]

\(\Xi \rightarrow j\): matrix of the predecessor LVs

\(\xi_j\): successor LV \(\Rightarrow\) Quantile Correlation

\[ qcov_\theta \left\{ \xi_j, \xi_j \rightarrow \right\} = \text{cov} \left\{ I \left( \xi_j \rightarrow - Q_\theta (\xi_j \rightarrow) > 0 \right), \xi_j \right\} \]

\[ qcor_\theta = \frac{qcov_\theta \left\{ \xi_j, \xi_j \rightarrow \right\}}{\sqrt{\text{var} \left\{ \psi_\theta (\xi_j \rightarrow - Q_\theta (\xi_j \rightarrow)) \right\} \text{var} (\xi_j)}} = \frac{qcov_\theta \left\{ \xi_j, \xi_j \rightarrow \right\}}{\sqrt{\left( \theta - \theta^2 \right) \text{var} (\xi_j)}} \]

\(-I\): indicator function

\(-Q_\theta (\xi_j \rightarrow)\): unconditional \(\theta^{th}\) quantile

\(-\xi_j \rightarrow\): a successor LV

\(-\psi_\theta (u) = \theta - I (u < 0)\)
Factorial and centroid scheme (quantile $\theta$)

$\xi_j :$ predecessor LV $\Rightarrow$ Quantile Correlation

$$qcov_\theta \{\xi_j, \xi_{j\rightarrow}\} = \text{cov} \{I (\xi_{j\rightarrow} - Q_{\theta}(\xi_{j\rightarrow}) > 0), \xi_j\}$$

$$qcor_\theta = \frac{qcov_\theta \{\xi_j, \xi_{j\rightarrow}\}}{\sqrt{(\theta - \theta^2) \text{var}(\xi_j)}}$$

The direction of the links in the structural model is taken into account!
QC-PM: the outer estimation

Outwards-directed

Inwards-directed

University of Padua

C.Davino

17th-18th September 2015

QC-PM FOR MEASURING WELL-BEING

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QC-PM: the outer estimation

Mode Q
an innovative outer estimation mode measured through QC

Mode $Q_{\text{Outwards-directed}}$

Mode $Q_{\text{Inwards-directed}}$
## The proposed QC-PMs

<table>
<thead>
<tr>
<th>Outer weights</th>
<th>Inner scheme</th>
<th>Path weighting</th>
<th>Factorial</th>
<th>Centroid</th>
<th>Quantile method.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outwards</td>
<td>QPLSPM1</td>
<td>QPLSPM2</td>
<td>QPLSPM3</td>
<td>Simple QR</td>
<td></td>
</tr>
<tr>
<td>Inwards</td>
<td>QPLSPM4</td>
<td>QPLSPM5</td>
<td>QPLSPM6</td>
<td>Multiple QR</td>
<td></td>
</tr>
<tr>
<td>Q&lt;sub&gt;OUTWARDS&lt;/sub&gt;</td>
<td>QPLSPM7</td>
<td>QPLSPM8</td>
<td>QPLSPM9</td>
<td>QC</td>
<td></td>
</tr>
<tr>
<td>Q&lt;sub&gt;INWARDS&lt;/sub&gt;</td>
<td>QPLSPM10</td>
<td>QPLSPM11</td>
<td>QPLSPM12</td>
<td>QC</td>
<td></td>
</tr>
<tr>
<td>Quantile method.</td>
<td>QR&amp;QC</td>
<td>QC</td>
<td>QC</td>
<td>QC</td>
<td></td>
</tr>
</tbody>
</table>
Data structure

Composite Indicator

Dimensions

Subdimensions

Indicators
## Data structure

### Dimensions

<table>
<thead>
<tr>
<th></th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>health</td>
</tr>
<tr>
<td>2</td>
<td>education and training</td>
</tr>
<tr>
<td>3</td>
<td>work and life balance</td>
</tr>
<tr>
<td>4</td>
<td>economic well-being</td>
</tr>
<tr>
<td>5</td>
<td>social relationships</td>
</tr>
<tr>
<td>6</td>
<td>politics and institutions</td>
</tr>
<tr>
<td>7</td>
<td>security</td>
</tr>
<tr>
<td>8</td>
<td>landscape and cultural heritage</td>
</tr>
<tr>
<td>9</td>
<td>environment</td>
</tr>
<tr>
<td>10</td>
<td>research and innovation</td>
</tr>
<tr>
<td>11</td>
<td>quality of services</td>
</tr>
</tbody>
</table>
Data structure

### Dimensions (sub-dimensions) [ # indicators ]

1. **health** (life expectancy, mortality) [8]
2. **education and training** (educational attainment, participation in education, competencies, lifelong learning) [8]
3. **work and life balance** (work participation, employment, unemployment, safety at work) [9]
5. **social relationships** (disability, immigration, civil society) [8]
6. **politics and institutions** (political participation, institutional representation, local government) [10]
7. **security** (crime rate, physical integrity) [6]
Data structure

### Dimensions (sub-dimensions) [# indicators]

<table>
<thead>
<tr>
<th>#</th>
<th>Description</th>
<th>Sub-dimensions</th>
<th># Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>landscape and cultural heritage</td>
<td>cultural heritage, landscape</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>environment</td>
<td>quality of environment, environmental resources consumption, environmental sustainability</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>research and innovation</td>
<td>innovation, research</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>quality of services</td>
<td>provision of public services, access to public services</td>
<td>7</td>
</tr>
</tbody>
</table>

### Data matrix

- 110 provinces
- 11 Dimensions
- 31 sub-dimensions
- 87 indicators
The BES structural model
Data pre-processing

- Indicator selection: not reliable indicators, related to districts, uncorrelated
- Missing data imputation: regional values
- Multicollinearity analysis: redundant indicators
- Preliminary PLSPM using Mode PLS: low weights, multidimensional blocks

Final data matrix

110 provinces
11 Dimensions
29 sub-dimensions
63 indicators
The BES hierarchical model
## The BES dataset

<table>
<thead>
<tr>
<th>HEALTH</th>
<th>LIFE EXPECTANCY</th>
<th>Life expectancy at birth (male)</th>
<th>I.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Life expectancy at birth (female)</td>
<td>I.2</td>
<td></td>
</tr>
<tr>
<td>MORTALITY</td>
<td>Infant mortality rate</td>
<td>I.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Avoidable mortality rate (0-74 years old)</td>
<td>I.8</td>
<td></td>
</tr>
<tr>
<td>EDUCATION AND TRAINING</td>
<td>EDUCATIONAL ATTAINMENT</td>
<td>Early leavers from education and training</td>
<td>II.1</td>
</tr>
<tr>
<td></td>
<td>People in working age with lower secondary education or less</td>
<td>II.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PARTICIPATION IN EDUCATION</td>
<td>Participation in upper secondary education</td>
<td>II.3</td>
</tr>
<tr>
<td></td>
<td>Participation in tertiary education (19-25 years old)</td>
<td>II.4</td>
<td></td>
</tr>
<tr>
<td>COMPETENCIES</td>
<td>Level of literacy</td>
<td>II.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level of numeracy</td>
<td>II.7</td>
<td></td>
</tr>
<tr>
<td>LIFELONG LEARNING</td>
<td>Participation in lifelong learning (25-64 years old)</td>
<td>II.8</td>
<td></td>
</tr>
<tr>
<td>WORK AND LIFE BALANCE</td>
<td>WORK PARTICIPATION</td>
<td>Non-participation rate (15-74 years)</td>
<td>III.1</td>
</tr>
<tr>
<td></td>
<td>Gender inequality in non-participation rate (M-F)</td>
<td>III.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EMPLOYMENT</td>
<td>Employment rate (20-64 years old)</td>
<td>III.3</td>
</tr>
<tr>
<td></td>
<td>Gender inequality in employment rate (M-F)</td>
<td>III.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UNEMPLOYMENT</td>
<td>Unemployment rate (15 – 74 years old)</td>
<td>III.6</td>
</tr>
<tr>
<td></td>
<td>SAFETY AT WORK</td>
<td>Incidence rate of occupational injuries, fatal or leading to permanent disability</td>
<td>III.9</td>
</tr>
</tbody>
</table>
### The BES dataset

<table>
<thead>
<tr>
<th>Economic Well-Being</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>Gross disposable income per household</td>
<td>IV.1</td>
</tr>
<tr>
<td></td>
<td>Average remuneration of employees (yearly)</td>
<td>IV.2</td>
</tr>
<tr>
<td></td>
<td>Annual pension income per capita (yearly average)</td>
<td>IV.4</td>
</tr>
<tr>
<td></td>
<td>Pensioners with a low pension income (&lt;500 euros)</td>
<td>IV.5</td>
</tr>
<tr>
<td>Wealth</td>
<td>Households assets (total amount in average)</td>
<td>IV.6</td>
</tr>
<tr>
<td>Inequality</td>
<td>Gender inequalities in average remuneration of employees (M-F)</td>
<td>IV.7</td>
</tr>
<tr>
<td></td>
<td>Age-groups inequalities in average remuneration of employees</td>
<td>IV.8</td>
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<tr>
<td>Financial Difficulties</td>
<td>Evictions from home per households</td>
<td>IV.9</td>
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<td></td>
<td>Households non performing loans (input flows)</td>
<td>IV.10</td>
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<tr>
<td>Social Relationships</td>
<td>Accessible school environments for disabled people</td>
<td>V.1-2-3</td>
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<tr>
<td>Disability</td>
<td>Disabled pupils enrolled</td>
<td>V.4</td>
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<tr>
<td>Immigration</td>
<td>Foreigners’ acquisitions of Italian citizenship</td>
<td>V.5</td>
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<tr>
<td>Civil Society</td>
<td>Social cooperatives</td>
<td>V.6</td>
</tr>
<tr>
<td></td>
<td>No-profit organizations</td>
<td>V.7</td>
</tr>
<tr>
<td></td>
<td>Volunteers in no-profit organizations (per 100 residents aged 14+)</td>
<td>V.8</td>
</tr>
<tr>
<td>Politics and Institutions</td>
<td>Electoral participation (European Parliament elections)</td>
<td>VI.1</td>
</tr>
<tr>
<td>Political Participation</td>
<td>Electoral participation (provincial elections)</td>
<td>VI.2</td>
</tr>
<tr>
<td>Institutional Representation</td>
<td>Women and political representation in municipalities</td>
<td>VI.3</td>
</tr>
<tr>
<td></td>
<td>Young people (&lt;40 years old) and political representation in municipalities</td>
<td>VI.4</td>
</tr>
<tr>
<td>Local Government</td>
<td>Municipalities: internal borrowing degree</td>
<td>VI.9</td>
</tr>
<tr>
<td></td>
<td>Municipalities: collecting capacity</td>
<td>VI.10</td>
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### The BES dataset

<table>
<thead>
<tr>
<th>SECURITY</th>
<th>CRIME RATE</th>
<th>Physical Integrity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>homicide rate</td>
<td>Road mortality index (killed per 100 road accidents)</td>
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<tr>
<td></td>
<td>Violent crimes reported</td>
<td>Road mortality index on rural roads (excluding motorways)</td>
</tr>
<tr>
<td>LANDSCAPE AND CULTURAL HERITAGE</td>
<td>Conservation of historic urban fabric</td>
<td>Presence of Historic Parks/Gardens and other Urban Parks recognized of significant public interest</td>
</tr>
<tr>
<td></td>
<td>Museums and similar institutions</td>
<td>Paying visitors of museums and similar institutions</td>
</tr>
<tr>
<td>CULTURAL HERITAGE</td>
<td>Urban green areas</td>
<td>Exceedings of the daily limit for the protection of human health for PM10 (Maximum number)</td>
</tr>
<tr>
<td>ENVIRONMENT</td>
<td>Hoseholds energy consumptions (electricity)</td>
<td>Drinkable water supplied every day per capita</td>
</tr>
<tr>
<td>QUALITY OF ENVIRONMENT</td>
<td>Urban cycle paths density</td>
<td>Energy from renewable sources (electricity)</td>
</tr>
<tr>
<td>RESOURCES CONSUMPTION</td>
<td>Landfill storage of waste</td>
<td></td>
</tr>
<tr>
<td>SUSTAINABILITY</td>
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<th>RESEARCH AND INNOVATION</th>
<th>INNOVATION</th>
<th></th>
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<tr>
<td>RESEARCH</td>
<td>Propensity to patent</td>
<td>X.1</td>
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<tr>
<td></td>
<td>New graduates in R&amp;D (residents - total)</td>
<td>X.6</td>
</tr>
<tr>
<td></td>
<td>Production industry specialization in knowledge-intensive sectors</td>
<td>X.7</td>
</tr>
<tr>
<td>QUALITY OF SERVICES</td>
<td>Irregularities in electricity supply</td>
<td>XI.1</td>
</tr>
<tr>
<td></td>
<td>Separate collection of urban waste</td>
<td>XI.3</td>
</tr>
<tr>
<td></td>
<td>Prisons overcrowding</td>
<td>XI.4</td>
</tr>
<tr>
<td></td>
<td>Urban public transport networks density</td>
<td>XI.6</td>
</tr>
<tr>
<td></td>
<td>Urban public transport capacity (seats per kilometers - rate per 1000 inhab.)</td>
<td>XI.7</td>
</tr>
<tr>
<td>PROVISION OF PUBLIC SERVICES</td>
<td>Taking charge of users for early childhood services</td>
<td>XI.2</td>
</tr>
<tr>
<td></td>
<td>Regional health service outflows (hospital admittance)</td>
<td>XI.5</td>
</tr>
</tbody>
</table>
### The proposed QC-PMs

<table>
<thead>
<tr>
<th>Outer weights</th>
<th>Path weighting</th>
<th>Factorial</th>
<th>Centroid</th>
<th>Quantile method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outwards</td>
<td>QPLSPM1</td>
<td>QPLSPM2</td>
<td>QPLSPM3</td>
<td>Simple QR</td>
</tr>
<tr>
<td>Inwards</td>
<td>QPLSPM4</td>
<td><strong>QPLSPM5</strong></td>
<td>QPLSPM6</td>
<td>Multiple QR</td>
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<tr>
<td>$Q_{OUTWARDS}$</td>
<td>QPLSPM7</td>
<td>QPLSPM8</td>
<td>QPLSPM9</td>
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</tr>
<tr>
<td>$Q_{INWARDS}$</td>
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<td>QPLSPM11</td>
<td>QPLSPM12</td>
<td>QC</td>
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<td>Quantile method.</td>
<td>QR&amp;QC</td>
<td>QC</td>
<td>QC</td>
<td></td>
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</tbody>
</table>
Health results

[Diagram showing various health-related topics interconnected, with "LIFE EXPECT." and "MORTALITY" highlighted.]
Health: PLSPM results

Path coefficients

<table>
<thead>
<tr>
<th>Life expectancy</th>
<th>Mortality</th>
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<tr>
<td>−0.6</td>
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<tr>
<td>−0.4</td>
<td>−0.2</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
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</table>

Outer weights

<table>
<thead>
<tr>
<th>I.1</th>
<th>I.2</th>
<th>I.3</th>
<th>I.8</th>
</tr>
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<tbody>
<tr>
<td>−0.4</td>
<td>0.2</td>
<td>0.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

I.1 Life expectancy at birth (male)
I.2 Life expectancy at birth (female)
I.3 Infant mortality rate
I.8 Avoidable mortality rate (0-74 years old)
Health: PLSPM and QC-PM results

Path coefficients

- life expectation
- mortality

Outer weights

C.Davino
17\textsuperscript{th}-18\textsuperscript{th} September 2015
Education and training results
Education and training: PLSPM results

Path coefficients

-0.4 0.0 0.4
attainment participation competencies lifelong l.

II.1 Early leavers from education and training
II.3 Participation in upper secondary education
II.6 Level of literacy
II.8 Participation in lifelong learning (25-64y)

Outer weights

II.1 II.2 II.3 II.4 II.6 II.7 II.8
-0.2 0.0 0.2 0.3

II.2 People in working age with lower secondary educ.
II.4 Participation in tertiary education (19-25y)
II.7 Level of numeracy
Education and training: PLSPM and QC-PM results

Path coefficients

Outer weights
BES: path coefficients

PLSPM

- Economic well-being
- Work and life balance
- Quality of services
- Politics and institutions
- Education
- Cultural heritage
- Research and innovation
- Health
- Social relationships
- Environment
- Security

0.00 0.02 0.04 0.06 0.08 0.10 0.12

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QC-PM FOR MEASURING WELL-BEING
BES and geographic areas

PLSPM - North west

path coefficients

LV averages

-1.0 -0.5 0.0 0.5 1.0
0.08 0.09 0.10 0.11 0.12 0.13 0.14

ECONOMIC WORK
QUALITY SERV
POLITICS
EDUCATION
LANDSCAPE
HEALTH
SOCIAL RELAT.
SECURITY
ENVIRONMENT
BES and geographic areas

PLSPM - North east

path coefficients

LV averages

ECONOMIC WORK
QUALITY SERV.
POLITICS
EDUCATION
LANDSCAPE
RESEARCH
HEALTH
SOCIAL RELAT.
SECURITY
ENVIRONMENT
BES and geographic areas

PLSPM - Center

path coefficients

ECONOMIC WELL-BEING
QUALITY SERV.
POLITICS
EDUCATION
RESEARCH
LANDSCAPE
HEALTH
SOCIAL RELAT.
ENVIRONMENT
SECURITY

LV averages

-1.0 -0.5 0.0 0.5 1.0
0.08 0.09 0.10 0.11 0.12 0.13 0.14
BES and geographic areas

PLSPM - South and islands

- Health
- Education
- Quality Service
- Politics
- Economic Well-Being
- Social Relations
- Security
- Environment
- Landscape

Path coefficients: -1.0 to 1.0
LV averages: 0.08 to 0.14
BES: path coefficients

PLSPM

Economic well-being
Work and life balance
Quality of services
Politics and institutions
Education
Cultural heritage
Research and innovation
Health
Social relationships
Environment
Security

0.00 0.02 0.04 0.06 0.08 0.10 0.12
BES: path coefficients

QC-PM: $\theta = 0.1$

- Economic well-being
- Work and life balance
- Quality of services
- Politics and institutions
- Education
- Cultural heritage
- Research and innovation
- Health
- Social relationships
- Environment
- Security

The chart shows the path coefficients for various domains, with Environment having the lowest coefficient and Security having the highest.
BES: path coefficients

QC-PM: $\theta = 0.25$

- Economic well-being
- Work and life balance
- Quality of services
- Politics and institutions
- Education
- Cultural heritage
- Research and innovation
- Health
- Social relationships
- Environment
- Security

Security: 0.08
Environment: 0.10
Social relationships: 0.12
Cultural heritage: 0.12
Education: 0.12
Research and innovation: 0.10
Health: 0.10
Politics and institutions: 0.12
Quality of services: 0.12
Work and life balance: 0.12
Economic well-being: 0.12
BES: path coefficients

QC-PM: $\theta=0.5$

- Economic well-being
- Work and life balance
- Quality of services
- Politics and institutions
- Education
- Cultural heritage
- Research and innovation
- Health
- Social relationships
- Environment
- Security

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QC-PM FOR MEASURING WELL-BEING
BES: path coefficients

QC-PM: $\theta = 0.75$

<table>
<thead>
<tr>
<th>Dimension</th>
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<tr>
<td>Work and life balance</td>
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</tr>
<tr>
<td>Quality of services</td>
<td>0.12</td>
</tr>
<tr>
<td>Politics and institutions</td>
<td>0.12</td>
</tr>
<tr>
<td>Education</td>
<td>0.08</td>
</tr>
<tr>
<td>Cultural heritage</td>
<td>0.09</td>
</tr>
<tr>
<td>Research and innovation</td>
<td>0.15</td>
</tr>
<tr>
<td>Health</td>
<td>0.12</td>
</tr>
<tr>
<td>Social relationships</td>
<td>0.06</td>
</tr>
<tr>
<td>Environment</td>
<td>0.12</td>
</tr>
<tr>
<td>Security</td>
<td>0.08</td>
</tr>
</tbody>
</table>
BES: indirect effects

PLSPM

- innovation
- provision of ps
- access to ps
- civil society
- competencies
- local govern.
- institut. represent.
- income
- sustainability
- life expectancy
- employment
- research
- lifelong learning
- wealth
- political part.
- immigration
- disability
- resources consump.
- participation in educ
- safety at work
- financial diffic.
- inequality
- work participation
- quality of env.
- educ. attainment
- unemployment
- physical integrity
- crime rate
- mortality

−0.06 −0.02 0.02 0.04 0.06 0.08
BES: indirect effects

QC-PM: $\theta=0.1$

- innovation
- provision of ps
- access to ps
- civil society
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QC-PM FOR MEASURING WELL-BEING
BES: indirect effects

QC-PM: $\theta=0.25$

- innovation
- provision of ps
- access to ps
- civil society
- competencies
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17th-18th September 2015
BES: indirect effects

QC-PM: $\theta=0.5$

- innovation
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BES: indirect effects

QC-PM: $\theta = 0.75$

- innovation
- provision of ps
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- mortality

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QC-PM FOR MEASURING WELL-BEING
BES: indirect effects

QC-PM: $\theta=0.9$

- innovation
- provision of ps
- access to ps
- civil society
- competencies
- local govern.
- institut. represent.
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- sustainability
- life expectancy
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- mortality

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### Concluding remarks

#### When to use what version - Inner scheme

- **Path weighting/factorial/centroid**: take into account the direction of the links

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### When to use what version - Outer weights

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<td></td>
</tr>
</tbody>
</table>

**Conceptual definition of the LVs**

- Inwards/Outwards weights: discrete MVs
- $Q_I / Q_O$: simultaneously favouring dependence and interdependence relationships
Further developments

1. **Data structure**
   - Investigating an alternative QC measure more suited in case of discrete values
   - Exploring QC-PM with highly collinear data
   - Analysing differences among the QC-PMs through simulated data

2. **Model assessment**
   - Building goodness of fit measures in outwards and inwards version

3. **Model validation**
   - Exploring a jackknife approach to evaluate the statistical significance of the QC-PM coefficients
   - Testing the statistical significance of the differences among QC-PM coefficients at different quantiles through interquantile regression (Gould, 1987)
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Main references


[http://www.besdelleprovince.it/](http://www.besdelleprovince.it/)


