A MEASURE OF WELL-BEING ACROSS EUROPEAN COUNTRIES: AN INTEGRATED DEA-ENTROPY APPROACH

Eugenia Nissi, Annalina Sarra
Department of Economics
University G.d’Annunzio Chieti -Pescara

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The analysis of quality of life and well-being is deemed an important area of economic science in view of its relevant implications in the political, social and economic spheres.

This paper analyses the well-being across 21 European countries via Data Envelopment Analysis (DEA) combined with Shannon’s entropy formula with the aim to propose an indicator of quality of life.
OUTLINE

- Aim
- HDI and Well Being
- DEA Framework
- DEA Vs Entropy approach
- Results and Conclusions
Recently, a series of measures of well-being that go “beyond GDP” have been proposed, both in the academic world and in policy circles.

The most popular example of a measure that extends beyond GDP is the Human Development Index (HDI), proposed by the United Nations Development Programme (UNDP).

The HDI hinges on the capability approach and measures the performance of countries in terms of their income, health and education.
Another example is the “Better Life Initiative” launched by the OECD in 2011 (OECD, Organization for Economic Cooperation And Development 2011).

Also the European Commission has stressed the urgent need to collect more and better information for measuring well-being and sustainable development in its Communication on “GDP and beyond” (European Commission 2009).

The European Statistical System is encouraging the collection of the information necessary to construct a multidimensional measure of the quality of life in its different member states (European Statistical System 2011).

All these initiatives have been spurred by the publication of the so-called Stiglitz–Sen–Fitoussi report in (2009)
Let us consider $n$ Decision Making Units (DMUs) under evaluation.

Each $\text{DMU}_j$ ($j = 1, 2, ..., n$) is assumed to produce the same $s$ outputs by using the same $i$ inputs. There is a wide variety of DEA models for assessing these units.

According to the classic DEA model, the evaluation of the technical efficiency of a DMU is addressed by a ratio of a weighted sum of outputs to a weighted sum of inputs, such that no DMU’s efficiency scores can exceed one (Charnes et al. 1978).

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The efficiency of the $j$-th DMU $E_j$ can be obtained by solving the following linear fractional CCR DEA formulation:

$$\text{Max } E_j = \frac{\sum_{s=1}^{S} u_s O_{sj}}{\sum_{i=1}^{I} v_i I_{ij}}$$

$$\text{s.t } \frac{\sum_{s=1}^{S} u_s O_{sn}}{\sum_{i=1}^{I} v_i I_{in}} \leq 1 \quad \forall \ n$$

$$u_s \geq 0 \quad \forall \ s \quad v_i \geq 0 \quad \forall \ i$$

where $s = 1, 2, ..., S$ are the inputs used, $i = 1, 2, ..., I$ the output variables and $j = 1, 2, ..., n$ the DMU's.

Furthermore, in this model specification, $O_{sj}$ denotes the amount of the $s$-th output for the $j$-th DMU, $I_{ij}$ is the amount of the $i$-th input for the $j$-th DMU while $u_s$ and $v_i$ are the weights assigned to the $s$-th output and $i$-th input, respectively, objectively determined.
The motivation behind using Shannon’s entropy based DEA is the follows:

1) The efficiency discriminatory powers of the DEA models under specific situations are different, and it is hard for us to choose a specific model for performance evaluation in a specific scenario.

2) Each of the proposed models evaluates DMUs efficiencies from a different perspective, and has some valuable advantages in the analysis which we could not ignore. Efficiency result of a DMU obtained from a special DEA model cannot replace the corresponding efficiency score calculated by another DEA model.

3) Any single DEA model has limited discriminatory power in efficiency evaluation problems, so it is suitable to integrate different DEA models into evaluation simultaneously.
DEA efficiencies are first calculated for all possible variable subsets and analyzed using Shannon’s entropy theory to calculate the degree of the importance of each subset.

Then we combine the obtained efficiencies and the degrees of importance to generate a Comprehensive Efficiency Score which can observably improve the discrimination of traditional DEA models.
In other terms, the units under considerations have been evaluated by a set of different models, say:

\[ M = \{ M_1, M_2, \ldots, M_k \} \]

and the efficiency scores presented in the matrix form:

\[
\begin{pmatrix}
  E_{11} & E_{12} & \ldots & E_{1k} \\
  E_{21} & E_{22} & \ldots & E_{2k} \\
  \vdots & \vdots & \ddots & \vdots \\
  E_{n1} & E_{n2} & \ldots & E_{nk}
\end{pmatrix}
\]

To obtain a more balanced ranking of DMU's, the efficiency scores of various DEA models have been combined using Shannon's entropy method and the degree of importance of each model calculated via some established steps (Soleimani-damaneh and Zarepisheh, 2009).
Step 1: normalization of the efficiency matrix by setting
\[ e_{jk} = \frac{E_{jk}}{\sum_{j=1}^{n} E_{jk}}, \quad k = 1, 2, \ldots, K \]

Step 2: calculate entropy
\[ f_k = -(\ln n)^{-1} \sum_{j=1}^{n} e_{jk} \ln(e_{jk}), \quad k = 1, 2, \ldots, K \]

Step 3: compute the degree of diversification of \( M_k \) as \( d_k = 1 - f_k, \quad k = 1, 2 \ldots, K \)

Step 4: Set \( W_k = \frac{d_k}{\sum_{k=1}^{K} d_k} \), \( k = 1, 2, \ldots, K \) such that \( \sum_{k=1}^{K} W_k = 1 \)

Step 5: calculate a comprehensive efficiency score as:
\[ \theta_j = \sum_{k=1}^{K} W_k e_{jk}, \quad j = 1, 2, \ldots, n \]
Adopting the above specified DEA-Shannon entropy integrated approach we derive the Comprehensive Efficiency Scores for the European OECD countries, selected for the present analysis.

A total of 21 output-oriented constant return to scale DEA models were constructed, choosing two input variables (environmental pollution and employment rate) and three output indicators (satisfaction with life, life expectancy and income).
According to the empirical findings:

- The best performance in terms of well-being is observed in the Northern European countries (Sweden, France, Norway, Germany, United Kingdom).

- Whereas the bottom positions in the ranking are occupied by Poland, Slovenia, Hungary, Greece and Czech Republic.

- Italy lies in the middle of ranking, with a better performance compared to other Mediterranean countries, like Spain, Portugal and Greece.
### Europe Map By CES

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Apply the proposed methodology to a more complete dataset

Evaluate the effectiveness of this integrated approach by a simulation study

To Compare the results with those obtainable via a bootstrap procedure.
References

Thank you for your attention!!